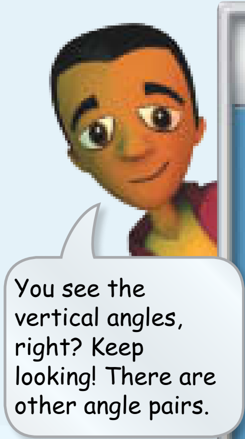
3-2 Properties of Parallel Lines

Common Core State Standards

G-CO.C.9 Prove theorems about lines and angles. Theorems include: . . . when a transversal crosses parallel lines, alternate interior angles are congruent . . .

MP 1, MP 3, MP 4, MP 6

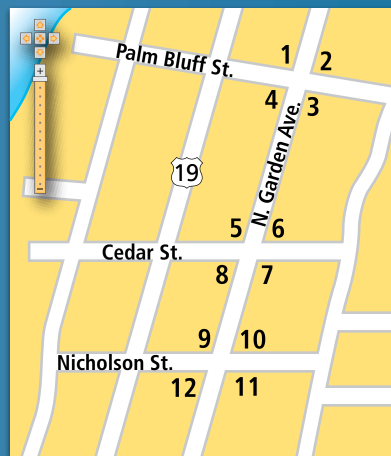
Objectives To prove theorems about parallel lines
To use properties of parallel lines to find angle measures



MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

Look at the map of streets in Clearwater, Florida. Nicholson Street and Cedar Street are parallel. Which pairs of angles appear to be congruent?

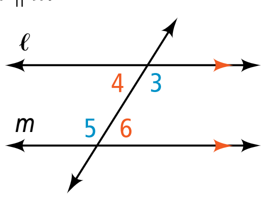


In the Solve It, you identified several pairs of angles that appear congruent. You already know the relationship between vertical angles. In this lesson, you will explore the relationships between the angles you learned about in Lesson 3-1 when they are formed by *parallel* lines and a transversal.

Essential Understanding The special angle pairs formed by parallel lines and a transversal are congruent, supplementary, or both.

take note

Postulate 3-1 Same-Side Interior Angles Postulate

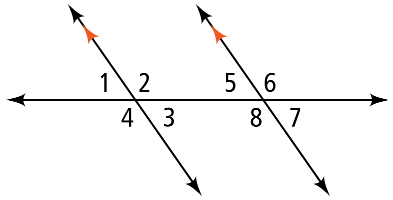
Postulate	If . . .	Then . . .
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.	$\ell \parallel m$ 	$m\angle 4 + m\angle 5 = 180$ $m\angle 3 + m\angle 6 = 180$

Problem 1 Identifying Supplementary Angles

Think

How do you find angles supplementary to a given angle?
Look for angles whose measure is the difference of 180 and the given angle measure.

The measure of ∠3 is 55. Which angles are supplementary to ∠3? How do you know?



By definition, a straight angle measures 180.

If $m\angle a + m\angle b = 180$, then $\angle a$ and $\angle b$ are supplementary by definition of supplementary angles.

$180 - 55 = 125$, so any angle x, where $m\angle x = 125$, is supplementary to $\angle 3$.

$m\angle 4 = 125$ by the definition of a straight angle.

$m\angle 8 = 125$ by the Same-Side Interior Angles Postulate.

$m\angle 6 = m\angle 8$ by the Vertical Angles Theorem, so $m\angle 6 = 125$.

$m\angle 2 = m\angle 4$ by the Vertical Angles Theorem, so $m\angle 2 = 125$.

Got It? **1. Reasoning** If you know the measure of one of the angles, can you always find the measures of all 8 angles when two parallel lines are cut by a transversal? Explain.

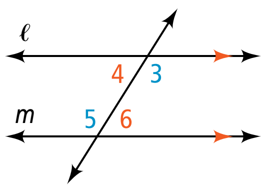
You can use the Same-Side Interior Angles Postulate to prove other angle relationships.

take note

Theorem 3-1 Alternate Interior Angles Theorem

Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

If . . .
$\ell \parallel m$

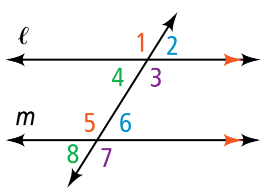


Then . . .
$\angle 4 \cong \angle 6$
$\angle 3 \cong \angle 5$

Theorem 3-2 Corresponding Angles Theorem

Theorem
If a transversal intersects two parallel lines, then corresponding angles are congruent.

If . . .
$\ell \parallel m$



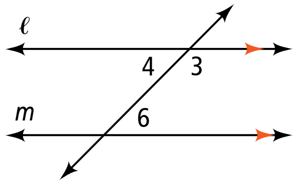
Then . . .
$\angle 1 \cong \angle 5$
$\angle 2 \cong \angle 6$
$\angle 3 \cong \angle 7$
$\angle 4 \cong \angle 8$

You will prove Theorem 3-2 in Exercise 25.

Proof Proof of Theorem 3-1: Alternate Interior Angles Theorem

Given: $\ell \parallel m$

Prove: $\angle 4 \cong \angle 6$

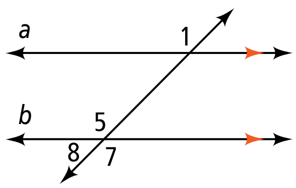


Statement	Reasons
1) $\ell \parallel m$	**1)** Given
2) $m\angle 3 + m\angle 4 = 180$	**2)** Supplementary Angles
3) $m\angle 3 + m\angle 6 = 180$	**3)** Same-Side Interior Angles Postulate
4) $m\angle 3 + m\angle 4 = m\angle 3 + m\angle 6$	**4)** Transitive Property of Equality
5) $m\angle 4 = m\angle 6$	**5)** Subtraction Property of Equality
6) $\angle 4 \cong \angle 6$	**6)** Definition of Congruence

Proof Problem 2 Proving an Angle Relationship

Given: $a \parallel b$

Prove: $\angle 1$ and $\angle 8$ are supplementary.



Know

- $a \parallel b$

From the diagram you know

- $\angle 1$ and $\angle 5$ are corresponding
- $\angle 5$ and $\angle 8$ form a linear pair

Need

$\angle 1$ and $\angle 8$ are supplementary, or $m\angle 1 + m\angle 8 = 180$.

Plan

Show that $\angle 1 \cong \angle 5$ and that $m\angle 5 + m\angle 8 = 180$. Then substitute $m\angle 1$ for $m\angle 5$ to prove that $\angle 1$ and $\angle 8$ are supplementary.

Statements	Reasons
1) $a \parallel b$	**1)** Given
2) $\angle 1 \cong \angle 5$	**2)** If lines are $\parallel$, then corresp. $\measuredangle$s are $\cong$.
3) $m\angle 1 = m\angle 5$	**3)** Congruent $\measuredangle$s have equal measures.
4) $\angle 5$ and $\angle 8$ are supplementary.	**4)** $\measuredangle$s that form a linear pair are suppl.
5) $m\angle 5 + m\angle 8 = 180$	**5)** Def. of suppl. $\measuredangle$s
6) $m\angle 1 + m\angle 8 = 180$	**6)** Substitution Property
7) $\angle 1$ and $\angle 8$ are supplementary.	**7)** Def. of suppl. $\measuredangle$s

Got It? 2. Using the same given information and diagram in Problem 2, prove that $\angle 1 \cong \angle 7$.

In the diagram for Problem 2, $\angle 1$ and $\angle 7$ are alternate exterior angles. In Got It 2, you proved the following theorem.

take note

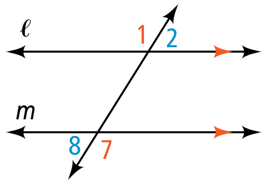
Theorem 3-3 Alternate Exterior Angles Theorem

Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

If . . .

$\ell \parallel m$



Then . . .

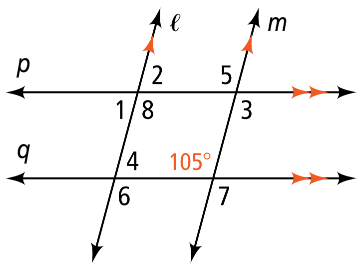
$\angle 1 \cong \angle 7$

$\angle 2 \cong \angle 8$

If you know the measure of one of the angles formed by two parallel lines and a transversal, you can use theorems and postulates to find the measures of the other angles.

Problem 3 Finding Measures of Angles

What are the measures of $\angle 3$ and $\angle 4$? Which theorem or postulate justifies each answer?



Think

How do $\angle 3$ and $\angle 4$ relate to the given 105° angle?

$\angle 3$ and the given angle are alternate interior angles. $\angle 4$ and the given angle are same-side interior angles.

Since $p \parallel q$, $m\angle 3 = 105$ by the Alternate Interior Angles Theorem.

Since $\ell \parallel m$, $m\angle 4 + 105 = 180$ by the Same-Side Interior Angles Postulate.
So, $m\angle 4 = 180 - 105 = 75$.

Got It? **3.** Use the diagram in Problem 3. What is the measure of each angle? Justify each answer.

a. $\angle 1$ **b.** $\angle 2$
c. $\angle 5$ **d.** $\angle 6$
e. $\angle 7$ **f.** $\angle 8$

You can combine theorems and postulates with your knowledge of algebra to find angle measures.

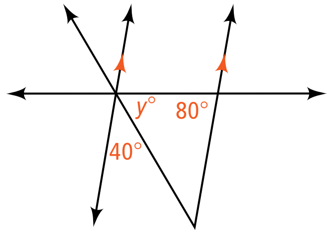
Think

What do you know from the diagram? You have one pair of parallel lines. The 80° angle and the angle formed by the 40° and y° angles are same-side interior angles.

Problem 4 Finding an Angle Measure

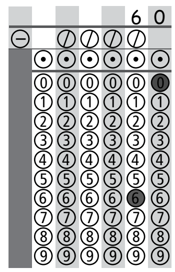
GRIDDED RESPONSE

Algebra **What is the value of y?**



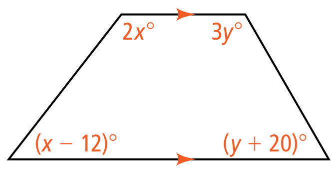
By the Angle Addition Postulate, $y + 40$ is the measure of an interior angle.

$(y + 40) + 80 = 180$	Same-side interior $\angle$s of $\parallel$ lines are suppl.
$y + 120 = 180$	Simplify.
$y = 60$	Subtract 120 from each side.



Got It? **4. a.** In the figure at the right, what are the values of x and y?

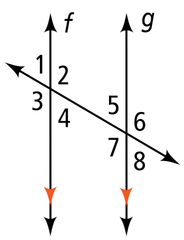
b. What are the measures of the four angles in the figure?



Lesson Check

Do you know HOW?

Use the diagram for Exercises 1–4.



1. Identify four pairs of congruent angles. (Exclude vertical angle pairs.)

2. Identify two pairs of supplementary angles. (Exclude linear pairs.)

3. If $m\angle 1 = 70$, what is $m\angle 8$?

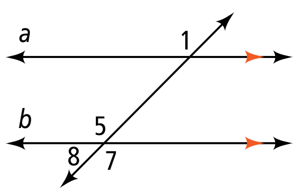
4. If $m\angle 4 = 70$ and $m\angle 7 = 2x$, what is the value of x?

Do you UNDERSTAND?

MATHEMATICAL PRACTICES

5. Compare and Contrast How are the Alternate Interior Angles Theorem and the Alternate Exterior Angles Theorem alike? How are they different?

6. In Problem 2, you proved that $\angle 1$ and $\angle 8$, in the diagram below, are supplementary. What is a good name for this pair of angles? Explain.



Practice and Problem-Solving Exercises

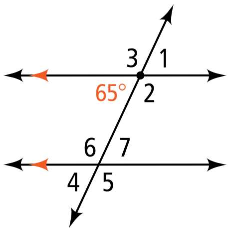
MATHEMATICAL PRACTICES

A Practice

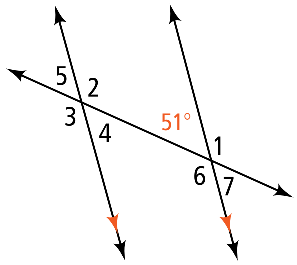
Identify all the numbered angles that are congruent to the given angle. Justify your answers.

See Problem 1.

7.



8.



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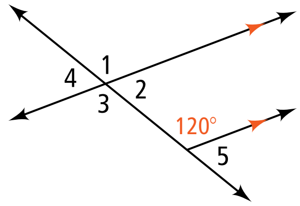
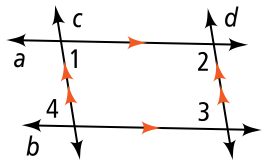


10. Developing Proof Supply the missing reasons in the two-column proof.

See Problem 2.

Given: $a \parallel b, c \parallel d$

Prove: $\angle 1 \cong \angle 3$



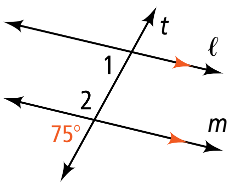
Statements	Reasons
1) $a \parallel b$	**1)** Given
2) $\angle 3$ and $\angle 2$ are supplementary.	**2) a.** ?
3) $c \parallel d$	**3)** Given
4) $\angle 1$ and $\angle 2$ are supplementary.	**4) b.** ?
5) $\angle 1 \cong \angle 3$	**5) c.** ?

11. Write a two-column proof for Exercise 10 that does not use $\angle 2$.
Proof

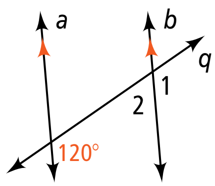
Find $m\angle 1$ and $m\angle 2$. Justify each answer.

See Problem 3.

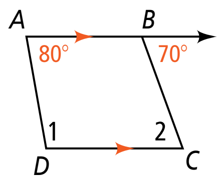
12.



13.



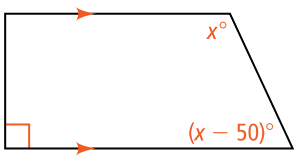
14.



Algebra **Find the value of x. Then find the measure of each labeled angle.**

See Problem 4.

15.



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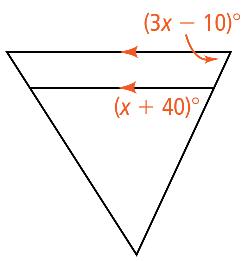
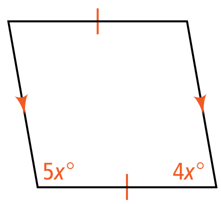


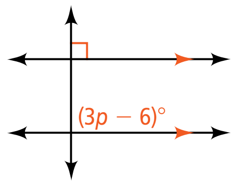
17.



B Apply

Algebra **Find the values of the variables.**

18.



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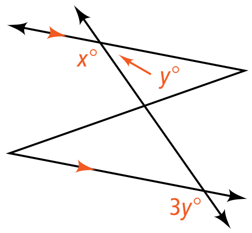
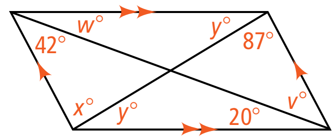


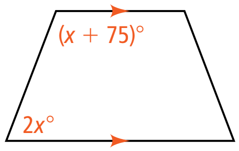
20.



21. Think About a Plan People in ancient Rome played a game called *terni lapilli*. The exact rules of this game are not known. Etchings on floors and walls in Rome suggest that the game required a grid of two intersecting pairs of parallel lines, similar to the modern game tick-tack-toe. The measure of one of the angles formed by the intersecting lines is 90°. Find the measure of each of the other 15 angles. Justify your answers.

- How can you use a diagram to help?
- You know the measure of one angle. How does the position of that angle relate to the position of each of the other angles?
- Which angles formed by two parallel lines and a transversal are congruent? Which angles are supplementary?

22. Error Analysis Which solution for the value of x in the figure at the right is incorrect? Explain.



A.

$$2x = x + 75$$
$$x = 75$$

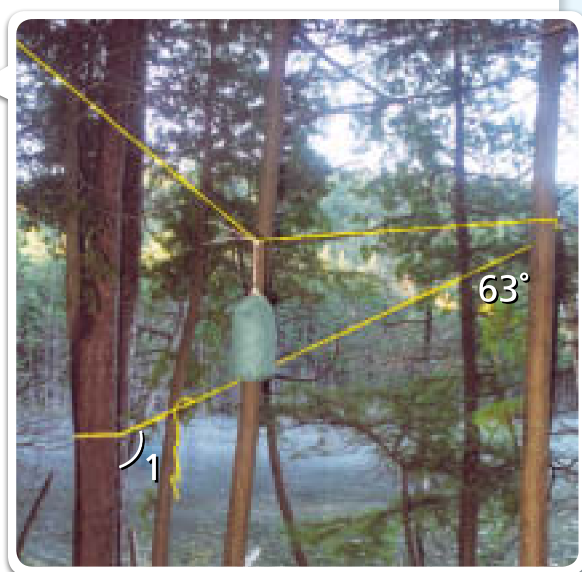
B.

$$2x + (x + 75) = 180$$
$$3x + 75 = 180$$
$$3x = 105$$
$$x = 35$$

23. Outdoor Recreation Campers often use a "bear bag" at night to avoid attracting animals to their food supply. In the bear bag system at the right, a camper pulls one end of the rope to raise and lower the food bag.

a. Suppose a camper pulls the rope taut between the two parallel trees, as shown. What is $m\angle 1$?

b. Are $\angle 1$ and the given angle *alternate interior angles*, *same-side interior angle*s, or *corresponding angles*?



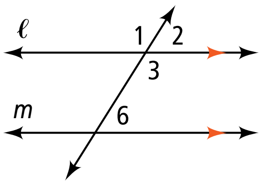
24. Writing Are same-side interior angles ever congruent? Explain.

25. Write a two-column proof to prove the Corresponding Angles Theorem (Theorem 3-2).

Proof

Given: $\ell \parallel m$

Prove: $\angle 2$ and $\angle 6$ are congruent.

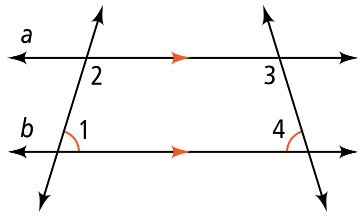


26. Write a two-column proof.

Proof

Given: $a \parallel b$, $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

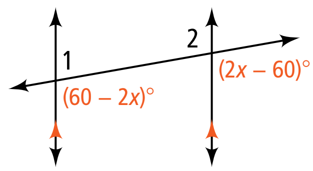


Challenge

Use the diagram at the right for Exercises 27 and 28.

27. **Algebra** Suppose the measures of $\angle 1$ and $\angle 2$ are in a 4 : 11 ratio. Find their measures. (Diagram is not to scale.)

28. **Error Analysis** The diagram contains contradictory information. What is it? Why is it contradictory?



PERFORMANCE TASK

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 6

Look back at the blueprint on page 139 of the plan for a city park. Choose from the following words, numbers, and expressions to complete the following sentences.

alternate interior	corresponding	same-side interior
alternate exterior	congruent	supplementary
vertical	59	90
95	121	$\angle 12$
$\angle 13$	$\angle 14$	$\angle 25$

The measure of $\angle 22$ is **a.** __?__, because alternate interior angles formed by parallel lines and a transversal are **b.** __?__.

The measure of $\angle 14$ is **c.** __?__, because **d.** __?__ angles formed by parallel lines and a transversal are congruent.

The measure of $\angle 32$ is **e.** __?__, because **f.** __?__ angles formed by parallel lines and a transversal are **g.** __?__.

The measure of **h.** __?__ is 17, because **i.** __?__ angles formed by parallel lines and a transversal are congruent.